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Supergravity duals of matrix string theory

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Abstract: We study holographic duals of type II and heterotic matrix string theories described by warped AdS_3 supergravities. By explicitly solving the linearized equations of motion around near horizon D-string geometries, we determine the spectrum of Kaluza-Klein primaries for type I, II supergravities on warped $AdS_3 \times S^7$. The results match those coming from the dual two-dimensional gauge theories living on the D-string worldvolumes. We briefly discuss the connections with the $\mathcal{N} = (8,8)$, $\mathcal{N} = (8,0)$ orbifold superconformal field theories to which type IIB/heterotic matrix strings flow in the infrared. In particular, we associate the dimension $(h,\bar{h}) = (3/2, 3/2)$ twisted operator which brings the matrix string theories out from the conformal point $(\mathbb{R}^8)^N/S_N$ with the dilaton profile in the supergravity background. The familiar dictionary between masses and "scaling" dimensions of field and operators are modified by the presence of non-trivial warp factors and running dilatons. These modifications are worked out for the general case of domain wall/QFT correspondences between supergravities on warped $AdS_{d+1} \times S^q$ geometries and super Yang-Mills theories with 16 supercharges.

Keywords: D-branes, M(atrix) Theories, AdS-CFT and dS-CFT Correspondence, Supergravity Models.

Contents

1. Introduction

Soon after Maldacena's proposal [1] for a holographic correspondence between string theory on anti-de Sitter spaces and conformal field theories living at the AdS boundaries, this correspondence was extended to more general string backgrounds and non-conformal field theories. In [2], a proposal relating string theory on near horizon Dp-brane geometries to $d = p+1$ dimensional super Yang-Mills (SYM) theories with sixteen supercharges was put forward. These so called domain wall/QFT dualities, later developed in [3, 4, 5], provide the simplest setting in which general ideas of holography can be tested in more interesting situations involving non-trivial warped geometries and running dilatons.

Despite its obvious interest, very few it is known at present about these correspondences. Various aspects were discussed in [3]; in particular, a series of potential dual gauged supergravities in various dimensions has been proposed which have not yet been systematically explored. In [5], two-point correlations functions for currents and stress energy tensors have been discussed with special emphasis on the cases $d = 5, 6$ and related brane-world scenarios. The aim of this paper is a systematic analysis of the spectrum of Kaluza-Klein (KK) supergravity harmonics and primary operators in the dual gauge theories for $d = 2$. This case is a particularly rich setting for a domain wall/QFT correspondence. It relates string theory on certain warped $AdS_3 \times S^7$ backgrounds to two-dimensional fundamental string or gauge theories, depending on whether we study systems of fundamental or D-strings. Fundamental string solutions are common to all five ten-dimensional string theories. The study of the domain wall/QFT correspondence in this case is especially interesting since the string background is free of RR fields. The case of D-strings is particular to type IIB and type I theory and will be the main subject of our investigation. The dual gauge theories in these cases correspond to the matrix string models [6, 7] proposed as non-perturbative definitions of type IIA and heterotic string theories. The correspondences under study here in principle provide a supergravity description of this physics (see [8] for early discussions on these ideas).

In this paper, we study supergravity duals of both matrix string descriptions. The relevant geometries involve warped $AdS_3 \times S^7$ spaces and running dilatons. By means of an harmonic analysis of the linearized equation of motions around the string backgrounds we determine the spectrum of primary fields in the corresponding AdS_3 supergravities. The results are shown to be in agreement with the dual description in terms of primary operators in the gauge theory. An important difference with pure AdS/CFT correspondences is the fact that the conformal group is no longer part of the background isometry group of this background which is rather given by the semidirect product of ISO$(1,1) \times$ SO(8) (two-dimensional Poincaré group and sphere isometries) with 16 supercharges. However, the full two-dimensional conformal algebra is realized as asymptotic conformal Killing isometries of the string background. In addition, these gauge theories are known to have an interesting IR dynamics governed by exactly solvable superconformal field theories (SCFTs) which can be thought of as second quantized type II or heterotic strings moving on $(\mathbb{R}^8)^N/S_N$ [6, 9]. Although the supergravity picture breaks down in this limit one can still hope to match some protected quantities like the BPS spectrum of states, two- and three-point functions, etc. in the supergravity side with those in the SCFT. We will give some evidence that this is indeed the case.

In the past, Kaluza-Klein spectra of supergravities have mainly been studied for reductions on factorized geometries. Together with [10], the reductions on warped $AdS_3 \times S^7$ presented here, to the best of our knowledge constitute the only examples in which the full KK spectrum has been worked out. We hope that the techniques developed here can help to improve this situation. As we will see, the familiar relations between masses and dimensions in pure AdS spaces get modified by the presence of warped factors and

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non-trivial dilatons. We work out the details of these modifications for the general case of domain wall/QFT correspondences between supergravity on warped $AdS_{d+1} \times S^q$ and SYM theories with 16 supercharges.

The paper is organized as follows: in section 2, we extend the familiar holographic relations between masses and conformal dimensions to the case of supergravities on warped $AdS_{d+1} \times S^q$. We present a sample calculation of the scalar two-point function in the warped AdS and discuss the conformal isometries of the warped string background. Sections 3, 4, and 5 are devoted to the study of KK harmonics of type IIA, IIB and type I supergravity, respectively, on warped $AdS_3 \times S^7$. We start by deriving the spectrum of sphere harmonics by a simple group theory analysis and confirm the full supermultiplet structure by explicit calculation of the linearized field equations. In section 6 we compare the supergravity results with the expectations from matrix string theory. In section 7 we comment on interesting directions of future research. The appendix combines a series of tables displaying the quantum numbers and structure of supermultiplets relevant for the discussions in the main text.

2. Warped $AdS_{d+1} \times S^q$ geometries

In this section we apply the ideas of [11] to extend the dictionary between masses and scaling dimensions on AdS spaces to domain wall/QFT correspondences involving warped $AdS_{d+1} \times S^q$ geometries. More specifically, we consider spaces described by a metric

$$d\hat{s}^2 = z^\omega \, ds^2 = z^\omega \left[\frac{\ell^2}{z^2} \, dx^\mu \, dx_\mu + \widetilde{\ell}^2 \, d\Omega_q \right] . \tag{2.1}$$

Greek letters $\mu = 0, 1, \ldots, d$, refer to components along AdS_{d+1} with $x^d \equiv z$, while Arabic indices $m = d+1, \ldots, D-1$ run over the S^q sphere with metric element $d\Omega_q$. Geometries of the general form (2.1) typically arise as near horizon geometries of p-brane solutions of low energy supergravities. In addition, they involve a non-trivial flux for a rank $d+1$ form and — unless $(D,p) = (10,3), (11,2), (11,5)$ — a running dilaton. For definiteness, let us consider Dp-branes, such that $D = 10$, $d = p+1$, $q = 9-d$, and (for $d \neq 6$)

$$\omega = -\frac{(d-4)^2}{4\,(6-d)} \,, \qquad \widetilde{\ell}^2 = \frac{\ell^2}{4}(6-d)^2 \,, \qquad z = \frac{2\sqrt{c_d \, g_{\rm YM}^2 \, N}}{6-d} \, r^{\frac{d-6}{2}} \,, \tag{2.2}$$

with a constant c_d, and r denoting the distance from the brane source. The Yang-Mills coupling constant $g_{\rm YM}$ is given by $g_{\rm YM}^2 = 2(2\pi)^{d-3} g_s (\alpha')^{\frac{d-4}{2}}$ and carries dimension of $[L]^{d-4}$. Perturbation theory is better organized in terms the dimensionless 't Hooft parameter

$$\lambda_{\rm eff} \equiv c_d \, g_{\rm YM}^2 \, N \left(\frac{r}{\alpha'} \right)^{d-4} . \tag{2.3}$$

The gauge theory is weakly coupled for $\lambda_{\rm eff} \ll 1$. The other two relevant parameters are

$$\ell^2 \, z^\omega \sim \sqrt{N} \, \lambda_{\rm eff}^{\frac{d-4}{4}} \,, \qquad e^\phi \sim \frac{\lambda_{\rm eff}^{\frac{8-d}{4}}}{N} \,, \tag{2.4}$$

denoting the AdS radius and the string coupling constant, cf. [2]. As usual, genus expansion corresponds to an expansion in $1/N$. We will always work in the limit of large N with $\lambda_{\rm eff}$ kept fixed where both supergravity and string perturbation can be trusted.

2.1 Scalar fields

In this and the next subsection we derive the relations between masses and scaling dimensions for fields moving on warped AdS geometries. For similar results on pure AdS spaces see [11, 12]. We start by considering a massless scalar field moving freely on (2.1). Rewriting the warped d'Alambertian in terms of the pure AdS_{d+1} one, the scalar equation of motion takes the form

$$\begin{aligned} z^\omega\, \hat{\Box}_D\, \phi &\equiv \frac{z^\omega}{\hat{e}}\, \partial_M \,\left(\hat{e}\, \hat{g}^{MN}\, \partial_N\, \phi\right) \\ &= \left(\Box_{AdS} + \frac{\omega}{2\,\ell^2}\,(D-2)\,\partial_z - m^2\right)\,\phi = 0\,. \end{aligned} \tag{2.5}$$

Here and in the following we denote by hats those quantities computed using the warped metric $\hat{g}_{MN} = z^\omega\, g_{MN}$ (2.1). The mass parameter m is defined by the eigenvalue equation:

$$\Box_{S^q}\,\phi = -m^2\,\phi\,,$$

and characterizes the harmonic mode of the scalar field ϕ along the sphere S^q. After a Fourier transform in the d-dimensional space spanned by $\mathbf{x} = (x^0, \ldots, x^{d-1})$

$$\phi(x^\mu) = \int d\mathbf{p}\, e^{i\mathbf{p}\,\mathbf{x}}\,\phi_p(z)\,, \tag{2.6}$$

the scalar equation (2.5) reduces to a second order differential equation for $\phi_p(z)$ of the kind

$$\left(z^2\,\partial_z^2 + (1-2\,a)\;z\,\partial_z + p^2\,z^2 + a^2 - \delta^2\right)\,\phi_p(z)\,, \tag{2.7}$$

with

$$a = \frac{d}{2} - \frac{\omega}{4}\,(D-2)\,, \qquad \delta = \sqrt{a^2 + m^2\ell^2}\,. \tag{2.8}$$

Its general solution can be expressed in terms of Bessel functions J_δ, Y_δ, as

$$\phi_p(z) = z^a\,\left[d_1\, J_\delta(pz) + d_2\, Y_\delta(pz)\right]. \tag{2.9}$$

The "conformal" dimension Δ of the holographically related operator $\mathcal{O}_\Delta$ can be extracted from the scaling behavior of the two solutions (2.9) for $\phi_p(z)$ at the boundary $z \sim 0$ as

$$\phi_p \sim \begin{cases} z^{E_0 - 2\,\delta}\,(\phi_{\rm def} + \mathcal{O}(z)) \\ z^{E_0}\,(\phi_{\rm vev} + \mathcal{O}(z)) \end{cases}\,, \tag{2.10}$$

with

$$\begin{aligned} E_0 &= a + \delta = a + \sqrt{a^2 + m^2\,\ell^2}\,, \\ \Delta &= \frac{E_0}{2}\,(6-d)\,. \end{aligned} \tag{2.11}$$

The last equation relates the quantum number E_0 which measures the power behavior in $z \sim r^{\frac{d-6}{2}}$ of ϕ near the boundary to the conformal dimension Δ associated to rescalings of the brane distance r. In the conformal case $\omega = 0$, $d = 4$, the two quantities coincide and we recover the familiar relation between scalar masses and conformal dimensions in AdS_5 with $a = \frac{d}{2} = 2$. In a general near horizon Dp-brane geometry one finds instead

$$a = \frac{8-d}{6-d}, \qquad \Delta = \frac{\widetilde{d}}{2} + \frac{1}{2}\sqrt{\widetilde{d}^2 + 4\,m^2\,\widetilde{\ell}^2} \tag{2.12}$$

where $\widetilde{d} = 8 - d$ denotes the worldvolume dimension of the magnetically dual brane. Note that for $d = 2$ and $d = 5$, these relations imply $a = \frac{d+1}{2}$, leading to a differential equation (2.7) which coincides with that for pure AdS space in dimension d+1. This reflects the fact that the near horizon geometries of fundamental strings and D4-branes of type IIA theory can be derived via dimensional reduction from those of M2 and M5 branes, respectively, a fact that will be extensively exploited later.

It is important to notice that the differential equation (2.7) keeps its form under rescaling of the field $\phi_p(z)$

$$\phi_p(z) \to z^{\xi}\phi_p(z)\,, \tag{2.13}$$

while the parameters get modified as $a \to a - \xi$, $\delta \to \delta$. Likewise, we note that the form of (2.7) remains invariant under

$$\phi_p(z) \to \phi_p(z) - \frac{z}{a \pm \delta}\,\phi_p'(z)\,, \tag{2.14}$$

under which its parameters transform as $a \to a - 1$, $\delta \to \delta \mp 1$.

2.2 Higher spin modes

Following [11, 12] one can easily extend the above dictionary between masses and conformal dimensions to higher spin fields. One starts by constructing Green functions $\Phi_0(z)$ defined as a solution of the field equations depending only on z (besides possible non trivial harmonic dependence on the sphere coordinates y which will be implicitly understood). The dependence on $\mathbf{x}$ can be restored later on by means of an SO$(1, d+1)$ transformation $x_\mu \to x_\mu/x^2$. Here we restrict ourselves to determined the z-dependence of the Green function associated i.e. the mode $\mathbf{p} = 0$.

For massless spin 1/2 fields $\hat{\Psi}_0(z)$ in D dimensions one finds

$$\begin{aligned} z^{\frac{\omega}{2}}\,\hat{\Gamma}^M\,\hat{D}_M\,\hat{\Psi}_0 &= \left(\Gamma^\mu\,D_\mu + \Gamma^m\,D_m + \frac{\omega}{4\,z}\,(D-1)\,\Gamma^z\right)\hat{\Psi}_0 \\ &= z^{-1}\,\Gamma^z\,\left(z\,\partial_z + (a - \frac{\omega}{4})\,\pm m\ell\right)\hat{\Psi}_0 \\ &= z^{-1+\frac{\omega}{4}}\,\Gamma^z\,\left(\partial_z + a\,\pm m\ell\right)\Psi_0 = 0\,, \end{aligned} \tag{2.15}$$

where in the last line we have introduced the rescaled field $\Phi \equiv z^{\omega/4}\,\hat{\Psi}$ in such a way that the value of a in (2.15) matches the one found for scalar fields in (2.8). The mass parameter m is again defined in terms of the sphere harmonics as

$$\Gamma^m D_m\,\Psi_0 = \pm m\,\Psi_0\,,$$

with SO(7) gamma matrices Γ^m. The scaling of the two solutions of (2.15) near the boundary $z \sim 0$ are

$$\Psi_0(z) \sim z^{a \pm m\ell} . \tag{2.16}$$

Using (2.10), we can identify $E_0 - 2\delta$ and E_0 as the powers of z from which we read off the mass/dimension relation

$$E_0 = a + |m\,\ell| , \qquad a = \frac{d}{2} - \frac{\omega}{4}\,(D-2) . \tag{2.17}$$

A similar analysis applies to higher spin modes. As an illustration of these cases let us consider a rank n form

$$\hat{A}_{\mu_1 \dots \mu_p m_{p+1} \dots m_n}(z) = e_{\mu_1}{}^{\alpha_1} \dots e_{m_n}{}^{a_n}\, A_{\alpha_1 \dots \alpha_p a_{p+1} \dots a_n}(z) , \tag{2.18}$$

with p legs along AdS_{d+1}. It is important to notice that unlike in more familiar AdS/CFT instances the scaling dimension of the dual operators associated to p forms in the warped AdS_{d+1} with different higher dimensional origins, are different due to extra ω-shifts coming from every leg aligned along the warped sphere. For simplicity, we restrict ourselves to the case where only one set of components $[\alpha_1 \dots \alpha_p a_{p+1} \dots a_n]$ with $\alpha_i \neq z$ is turned on. To derive the asymptotic behavior, we start from the ansatz

$$A_{\alpha_1 \dots \alpha_p a_{p+1} \dots a_n}(z) = c\, z^\lambda\, \epsilon_{\alpha_1 \dots \alpha_p a_{p+1} \dots a_n} . \tag{2.19}$$

The equations of motion

$$\partial_{M_0} \left(\hat{e}\, \hat{g}^{M_0 N_0} \dots \hat{g}^{M_n N_n}\, \partial_{[N_0}\, \hat{A}_{N_1 \dots N_n]} \right) = 0 , \tag{2.20}$$

then reduce to a quadratic equation in λ with solutions $\lambda_+ = E_0$, $\lambda_- = E_0 - 2\,\delta$ and

$$\begin{aligned} E_0 &= a + \delta = a + \sqrt{\left(a - p + \frac{n\,\omega}{2}\right)^2 + m^2\,\ell^2} , \\ a &= \frac{d}{2} - \frac{\omega}{4}\,(D-2) , \end{aligned} \tag{2.21}$$

where the mass parameter m is defined by the harmonic equation

$$D_m\, H^{m M_2 \dots M_n} = -m^2\, A^{M_2 \dots M_n} .$$

The relations (2.12), (2.17), (2.21) generalize the familiar relations [12] between masses of fields on AdS and scaling dimensions of the dual operators to domain wall/QFT involving warped geometries. In the next sections we apply this dictionary to the study of KK reductions of ten-dimensional strings on near horizon string-like backgrounds.

2.3 Scalar two-point function

The meaning of the quantum number E_0 (to which we will refer as the energy or scaling dimension) in the non-conformal instances that we study here, at first sight may seem obscure. As we mentioned in the introduction (see also section 2.4 below), the AdS group acts as the group of conformal isometries of the domain wall background and therefore gauge/supergravity quantities must transform covariant under conformal rescalings. It is natural to assign to supergravity states the quantum number E_0 to describe these scaling properties. Here we present a more technical definition of this quantity which makes contact with the definition of conformal dimensions in more familiar AdS/CFT instances. More precisely, we show that, under suitable normalizations, the operator two-point function derived from supergravity scalar fields on warped AdS behaves like $\langle\mathcal{O}(\mathbf{x})\mathcal{O}(\mathbf{y})\rangle \sim |\mathbf{x}-\mathbf{y}|^{-2E_0}$. This is the expected result for a two-point function involving operators of dimension E_0 in a dual scale invariant boundary theory.

The derivation of the scalar two-point function in warped AdS is a straightforward generalization of the similar calculation in pure AdS (see for example [13, the appendix]). The action for a free massive scalar on warped AdS_{d+1} reads

$$\begin{aligned} S &= \frac{1}{2}\int d^{d+1}x\,\hat{e}\left(\hat{g}^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + m^2 z^{-\omega}\phi^2\right) \\ &= \frac{1}{2}\int d^{d+1}x\,e\,z^{\frac{\omega(d-1)}{2}}\left(g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + m^2\phi^2\right). \end{aligned} \tag{2.22}$$

The equations of motion coming from (2.22) were solved in the previous section. After Wick rotation and choosing appropriate boundary conditions they can be written as

$$\phi(x^\mu) = \int d\mathbf{p}\,e^{i\mathbf{p}\,\mathbf{x}}\,\phi_p(z) = \int d\mathbf{p}\,e^{i\mathbf{p}\,\mathbf{x}}\,\phi_B(p)\,K_\delta^\epsilon(z|p)\,, \tag{2.23}$$

with the renormalized Bessel function

$$K_\delta^\epsilon(z|p) \equiv \left(\frac{z}{\epsilon}\right)^a \frac{K_\delta(z|p)}{K_\delta(\epsilon|p)}\,,$$

satisfying

$$\lim_{z\to\epsilon} K_\delta^\epsilon(z|p) = 1\,, \qquad \lim_{z\to\infty} K_\delta^\epsilon(z|p) = 0\,.$$

Plugging (2.23) into the action one finds after integration by parts:

$$S = \frac{1}{2}\int d\mathbf{p}d\mathbf{p}'\,\delta(\mathbf{p}+\mathbf{p}')\,\phi_B(\mathbf{p})\phi_B(\mathbf{p}')\,\epsilon^{-2a+1}\,\lim_{z\to\epsilon}\partial_z\,K_\delta^\epsilon(z|p)\,,$$

from which one reads the two-point function (see [13] for details):

$$\langle\mathcal{O}(\mathbf{p})\mathcal{O}(\mathbf{p}')\rangle = \delta(\mathbf{p}+\mathbf{p}')\,p^{2\delta}\,\epsilon^{2(\delta-a)}\,2^{-2\delta}(-2\delta)\frac{\Gamma(1-\delta)}{\Gamma(1+\delta)}\,. \tag{2.24}$$

After Fourier transform one is finally left with

$$\langle\mathcal{O}(\mathbf{x})\mathcal{O}(\mathbf{y})\rangle \sim |\mathbf{x}-\mathbf{y}|^{-2\left(\delta+\frac{1}{2}d\right)}\,. \tag{2.25}$$

Notice that the dimension read off from (2.25) matches E_0 in (2.11) after a suitable rescaling of $\phi_p(z)$. As one can see from (2.13), such rescalings shift E_0 while keeping the argument under the square root in δ invariant. The whole information about the warped geometry is contained in this δ. In particular, the mass bound at which the argument of the square root

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becomes negative is shifted in the warped geometry with respect to its pure AdS cousin. It would be nice to supply this observation with a stability analysis in the spirit of [14]. A rather non-trivial consistency check of the whole picture follows from the need of a certain conspiracy between masses and warped factors entering δ in (2.11) in order to produce rational numbers as output of the square root. We will verify this by explicit calculations of the spectrum of harmonics in warped $AdS_3 \times S^7$. In [5], two-point correlation functions have been determined by a similar computation for the SO(8) current and the stress-energy tensor, leading to $\delta = 1/2$, and $\delta = 3/2$, respectively. As we will see below — cf. (4.24) and the subsequent discussion — these values are recovered in our general analysis; current and stress-energy tensor couple to the $n = 0$ components of spin $s_0 = 1, 2$ in table 6 of the appendix.

It is worth to spend some words on our choice of normalizations. As we have seen in our sample calculation, an overall shift in the definition of E_0 can be adjusted by a choice of frame. This is clear from the fact that since the dilaton background carries an explicit z-dependence, fields which are redefined by a dilaton-dependent rescaling couple to operators of different dimensions. In particular, fluctuations of the metric in the Einstein or string frame carry different energies E_0. There is no a priori privileged choice. We choose normalizations in such a way that all field equations reduce to scalar type of equations with $E_0 = a + \delta$ and a given by (2.8). In the warped AdS_3 case this correspond to choose $a = 3/2$. This choice is natural for the type IIA case where string solutions descending from M2 branes by dimensional reduction are associated to a $d = 3$ SCFT. For the sake of comparison we adopt the same normalization in the type IIB and type I case where no analog of the higher dimensional origin is available.

2.4 $AdS_3 \times S^7$ vacua and conformal Killing isometries

We will focus on string like solutions with near horizon geometries conformal to $AdS_3 \times S^7$, (for earlier investigations of these backgrounds, see e.g. [15]). These vacuum configurations involve a non-trivial metric g_{MN}, the dilaton ϕ and a rank three form H_3 given by (here and in the following we always work in the Einstein frame):

$$\begin{aligned} d\hat{s}^2 &= z^\omega \, ds^2 = z^\omega \left[\frac{\ell^2}{z^2} \left(-2\, dx_+ \, dx_- + dz^2\right) + 4\, \ell^2 \, d\Omega_7 \right] \\ e^{\epsilon\phi} &= \Phi_0 \, z^{3/2} \\ H_{01z} &= \frac{3\, \ell^2}{z^4} \end{aligned} \tag{2.26}$$

with dimensionful constant Φ_0, a sign $\epsilon = -1, 1$ for the fundamental and D-string, respectively, and $\omega = -1/4$. The variable z is related to the D-brane distance r via $z = r^{-2}$ and therefore conformal dimensions Δ are given in terms of the E_0 scaling dimension defined through (2.11) by

$$\Delta = 2\, E_0 \, . \tag{2.27}$$

The string vacuum preserves sixteen supercharges. This can be easily understood by noticing that (2.26) arises from dimensional reduction along AdS_4 of the maximal supersymmetric $AdS_4 \times S^7$ 11d-vacuum. According to [16], half of the AdS_4 Killing spinors are preserved by such reductions.

The geometry (2.26) is also invariant under $\mathrm{ISO}(1,1)\times\mathrm{SO}(8)$ with $\mathrm{SO}(8)$ the isometries of the seven sphere and $\mathrm{ISO}(1,1)$ the two-dimensional Poincaré group. In addition one can verify the invariance under the following rescaling

$$x_\mu \to \lambda^2\, x_\mu\,, \qquad \ell^2 \to \lambda^{-2\,\omega}\,\ell^2\,. \tag{2.28}$$

This is clearly not a symmetry of the theory since it relates the physics at a given AdS radius ℓ^2 to that at $\lambda^{-2\,\omega}\,\ell^2$. The existence of this invariance however requires that quantities in the dual QFT should transform covariantly under conformal rescalings. It is interesting to notice that the rescalings (2.28) leave invariant the effective 't Hooft parameter $\lambda_{\rm eff}$.

One can still go further and check that the full two-dimensional conformal algebra arises as asymptotic conformal Killing isometries of the warped geometry (2.26). The algebra is realized near the boundary $z \sim 0$ (up to $\mathcal{O}(z^4)$ terms) in terms of the following two sets of Virasoro generators

$$\begin{aligned}
L_n &= x_+^{n+1}\,\partial_+ + \frac{1}{4} z^2\,\ell^2\,x_+^{n-1}\,n(n+1)\,\partial_- + \frac{1}{2}(n+1)\,x_+^n\,z\,\partial_z\,,\\
\bar{L}_n &= x_-^{n+1}\,\partial_- + \frac{1}{4} z^2\,\ell^2\,x_-^{n-1}\,n(n+1)\,\partial_+ + \frac{1}{2}(n+1)\,x_-^n\,z\,\partial_z\,,
\end{aligned} \tag{2.29}$$

which satisfy the conformal Killing equations

$$\nabla_{(\mu}\,\xi_{\nu)} - \frac{1}{2}\,g_{\mu\nu}\nabla^\rho\,\xi_\rho = \mathcal{O}(z^\omega)\,, \tag{2.30}$$

up to terms of order $\mathcal{O}(z^\omega)$ which at $z \sim 0$ fall off much faster than the metric. Among these generators one can easily identify the $\mathrm{ISO}(1,1)$ Killing algebra with spin and translations realized by $s_0 = L_0 - \bar{L}_0$, and $L_{-1}, \bar{L}_{-1}$, respectively. Finally, the scaling properties of fields/operators will be described by $E_0 = L_0 + \bar{L}_0 = z\partial_z$, which is clearly not a Killing but a conformal Killing vector of (2.26).

The realization of the full two-dimensional conformal algebra near the boundary $z \sim 0$ suggests that this string background represents some sort of non-conformal deformation of a more fundamental dual SCFT living at the AdS_3 boundary. The natural candidates for such dual theories in the case of D-strings are the orbifold SCFT's [6, 9] to which the two-dimensional gauge theories governing the dynamics of D-strings in type IIB and type I theory flow in the infrared. They can be thought as second quantized type II or heterotic strings moving on $(\mathbb{R}^8)^N/S_N$. Some striking evidence for such a correspondence can be already be observed from the scaling behavior of the dilaton in the string vacuum (2.26)

$$e^\phi = \Phi_0\,z^{3/2}\,, \tag{2.31}$$

implying that the associated operator is a scalar of conformal dimension $\Delta = 2E_0 = 3$. This suggests to identify this operator with the one responsible for bringing the SCFT away from the conformal point and identified in [6] as the $\mathbb{Z}_2$ twist field with dimensions $(h,\bar{h}) = (3/2, 3/2)$, i.e. $s_0 = 0$ and $\Delta = 3$. In this picture, the domain wall solution can be seen as a deformation via the expectation value g_s of a somewhat mysterious conformal point.

3. Type IIA supergravity on warped $AdS_3 \times S^7$

In this section, we compute the Kaluza-Klein spectrum of type IIA supergravity on the warped $AdS_3 \times S^7$ background. We first perform a simple group theory analysis to determine the spectrum of sphere harmonics and applies to the type IIB case as well. To further obtain the masses of the fields in the sense discussed in the last section, we employ the $AdS_4 \times S^7$ vacuum of eleven-dimensional supergravity whose spectrum has extensively been studied in the eighties [17], and from which the warped $AdS_3 \times S^7$ spectrum of type IIA descends upon further dimensional reduction.

3.1 Group theory and sphere harmonics

The spectrum of $\mathrm{SO}(q+1)$ representations appearing in the Kaluza-Klein reduction of a D-dimensional supergravity on the sphere S^q is essentially determined by group theory [18] (see [19] for a recent discussion in a similar context). The harmonic expansion of a field transforming in the $\mathcal{R}_{\mathrm{SO}(q)}$ representation of the Lorentz group of S^q comprises all the representations $\mathcal{R}_{\mathrm{SO}(q+1)}$ of the isometry group $\mathrm{SO}(q+1)$ that contain $\mathcal{R}_{\mathrm{SO}(q)}$ in their decomposition. The presence of a warp factor and a running dilaton does not alter this analysis. In this section we apply this group theory analysis to derive the spectrum of SO(8) representations in the type IIA string background.

The on-shell field content of ten-dimensional type IIA supergravity $(\mathbf{8_v}+\mathbf{8_s})\times(\mathbf{8_v}+\mathbf{8_c})$ decomposes under the SO(7) Lorentz group of S^7 as

$$\mathcal{R}^{\mathrm{II}}_{\mathrm{SO}(7)} = 3\cdot\mathbf{1} + 3\cdot\mathbf{7} + 4\cdot\mathbf{8} + 2\cdot\mathbf{21} + \mathbf{27} + \mathbf{35} + 2\cdot\mathbf{48}\,. \tag{3.1}$$

Every representation in (3.1) gives rise to a tower of harmonics of those SO(8) representations in which it is contained upon decomposition under SO(7). We have summarized these series in table 1. Together, this gives the SO(8) field content of type IIA supergravity on $AdS_3 \times S^7$ summarized in table 2.

SO(7)	in SO(8)	
1	$(n,0,0,0)$	$n=0,1,\ldots$
7	$(n,0,0,0)$	$n=1,2,\ldots$
	$(n,1,0,0)$	$n=0,1,\ldots$
21	$(n,0,1,1)$	$n=0,1,\ldots$
	$(n,1,0,0)$	$n=0,1,\ldots$
27	$(n,0,0,0)$	$n=2,3,\ldots$
	$(n,1,0,0)$	$n=1,2,\ldots$
	$(n,2,0,0)$	$n=0,1,\ldots$
8	$(n,0,1,0)+(n,0,0,1)$	$n=0,1,\ldots$
35	$(n,0,2,0)+(n,0,0,2)$	$n=0,1,\ldots$
	$(n,0,1,1)$	$n=0,1,\ldots$
48	$(n,0,1,0)+(n,0,0,1)$	$n=1,2,\ldots$
	$(n,1,1,0)+(n,1,0,1)$	$n=0,1,\ldots$

Table 1: Embedding of SO(7) representations into SO(8).

SO(8)	$n=0$	$n=1$	$n>1$
$(0,0,0,n)$	3	6	7
$(0,1,0,n)$	5	6	6
$(1,0,1,n)$	3	3	3
$(0,0,1,n)$	4	6	6
$(1,0,0,n)$	4	6	6
$(0,2,0,n)$	1	1	1
$(0,0,2,n)$	1	1	1
$(2,0,0,n)$	1	1	1
$(0,1,1,n)$	2	2	2
$(1,1,0,n)$	2	2	2

Table 2: SO(8) field content.

This spectrum may be conveniently organized in terms of supermultiplets of the superalgebra of background isometries, which as discussed above is build from the semidirect product of ISO(1,1) × SO(8) and 16 real supercharges. Moreover, we may even assign to all the fields quantum numbers of the conformal isometry group $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{OSp}(8|2,\mathbb{R})$. To analyze this multiplet structure it turns out to be useful to employ the representation structure of the well known spectrum of eleven-dimensional supergravity on S^7.

3.2 Linearized equations of motion via reduction from AdS_4

The warped $AdS_3 \times S^7$ string solution of type IIA supergravity descends from the much better studied $AdS_4 \times S^7$ vacuum of eleven-dimensional supergravity upon dimensional reduction along one of the AdS_4 coordinates. The spectrum of KK harmonics and linearized equations of motion around the type IIA string background can then easily be derived from the AdS_4 results [17] upon a further dimensional reduction. We focus on bosonic fluctuations. Field equations for scalar, vector and spin-2 fields in AdS_4 take the form

$$\Box\,\phi = \frac{1}{\tilde{\ell}^2}(\lambda-8)\,\phi\,, \tag{3.2}$$

$$D^M F_{MN} = \frac{1}{\tilde{\ell}^2}\lambda\,A_N\,, \tag{3.3}$$

$$\Box\,\eta_{MN} = \frac{1}{\tilde{\ell}^2}(\lambda-16)\,\eta_{MN}\,, \qquad \left(\eta^M{}_M = 0\,, \quad D^M\eta_{MN}=0\right). \tag{3.4}$$

with capital indices $M = \mu, y$, running over the 4d spacetime, and $\mu = 0,1,2$. The spectrum of masses λ has been derived in [17]. It is organized in supermultiplets of the group $\mathrm{OSp}(8|4,\mathbb{R})$. Upon dimensional reduction in y, the scalar equation (3.2) reduces to

$$\Box_{AdS_3}\left(z^{-1/2}\,\phi\right) = \frac{1}{\tilde{\ell}^2}(\lambda-3)\,z^{-1/2}\,\phi\,. \tag{3.5}$$

Evaluating this equation with the ansatz (2.6) $\phi(x^\mu) = e^{i\mathbf{p}\mathbf{x}}\,\phi_p(z)$, this leads to a three-

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dimensional equation of the type (2.7), with parameters

$$a = \frac{3}{2}\,, \qquad \delta = \frac{1}{2}\sqrt{1+\lambda}\,, \qquad E_0 = \frac{3}{2} + \frac{1}{2}\sqrt{1+\lambda}\,, \tag{3.6}$$

whose solution may be given in terms of Bessel functions. Notice that according to our discussion above, a differential equation with these characteristics coincides with the one coming from KK reduction of massless fields on warped $AdS_3 \times S^7$, cf. (2.8) with $\omega = -1/4$, $D = 10$. This is clear from the fact that these results could be derived directly from reduction of type IIA on the warped string background. In the following, we will choose (3.5) as the standard equation for scalar fields on the warped AdS_3 background.

The four-dimensional vector equation (3.3) with the ansatz $A_M = (A_\mu, e_y{}^y A_y) = (A_\mu, \frac{\ell}{z} A_y)$ turns into

$$\begin{aligned} \Box_{AdS_3}\left(z^{-1/2}\, A_y\right) &= \frac{1}{\tilde{\ell}^2}(\lambda-3)\, z^{-1/2}\, A_y\,, \\ D^\mu\left(z^{-1}\, F_{\mu\nu}\right) &= \frac{1}{\tilde{\ell}^2}\lambda\, z^{-1}\, A_\nu\,, \end{aligned} \tag{3.7}$$

on AdS_3. As expected, the component A_y decouples and gives rise to a scalar equation whose form coincides with (3.6). The three-dimensional vector equation may again be solved in terms of Bessel functions as

$$A_\mu = (A_+, A_-, A_z) = \left(\frac{p_+}{z}\left(a_1 - \frac{1}{2}z a_1' + a_2\right),\, \frac{p_-}{z}\left(a_1 - \frac{1}{2}z a_1' - a_2\right),\, p_+ p_- w\right), \tag{3.8}$$

where $a_{1,2}$ are solutions of the scalar field equation (2.7) with parameters (3.6). The energy may be read off from the scaling behavior of the flat components $\widetilde{A}_\pm = e_\pm{}^\mu\, A_\mu \sim z^{E_0}$ in the limit $z \sim 0$, cf. [11], and coincides with that of the scalar component (3.7). As expected, a four-dimensional vector on AdS upon dimensional reduction hence gives rise to a three-dimensional scalar and the two degrees of freedom of a massive spin 1 field, sharing the same energy E_0. Accordingly, we will assign to these fields quantum numbers $(\ell_0, \bar{\ell}_0)$ of the three-dimensional conformal isometry group SO(2, 2) as

$$\left(\frac{1}{2}E_0, \frac{1}{2}E_0\right), \qquad \left(\frac{1}{2}E_0 \mp \frac{1}{2}, \frac{1}{2}E_0 \pm \frac{1}{2}\right), \tag{3.9}$$

with $E_0 = \frac{3}{2} + \frac{1}{2}\sqrt{1+\lambda}$. Similarly, the reduction of the spin-2 field equations (3.4) yields a coupled system of a spin-0, spin-1 and spin-2 fields in warped AdS_3. At $\mathbf{p} = 0$ we find again scalar equations of the standard form (2.8) with characteristics (3.6). I.e. this field gives rise to three-dimensional fields with SO(2, 2) quantum numbers

$$\left(\frac{1}{2}E_0, \frac{1}{2}E_0\right), \qquad \left(\frac{1}{2}E_0 \mp \frac{1}{2}, \frac{1}{2}E_0 \pm \frac{1}{2}\right), \qquad \left(\frac{1}{2}E_0 \mp 1, \frac{1}{2}E_0 \pm 1\right). \tag{3.10}$$

Supplying the fields in table 2 with quantum numbers according to (3.9), (3.10) suggests a grouping into the $\mathcal{N} = (8, 8)$ supermultiplets $(\mathbf{n000})_{\mathrm{IIA}}$ defined in table 3. The state in the upper left corner has $(\ell_0, \bar{\ell}_0) = (\frac{1}{4}(n+2), \frac{1}{4}(n+2))$. The 8 unbroken supersymmetry

$(n{+}2000)$	$(n{+}1010)$	$(n100)$	$(n001)$	$(n000)$
$(n{+}1010)$	$(n100){+}(n020)$	$(n001){+}(n{-}1110)$	$(n000){+}(n{-}1011)$	$(n{-}1010)$
$(n100)$	$(n001){+}(n{-}1110)$	$(n000){+}(n{-}1011){+}(n{-}2200)$	$(n{-}1010){+}(n{-}2101)$	$(n{-}2100)$
$(n001)$	$(n000){+}(n{-}1011)$	$(n{-}1010){+}(n{-}2101)$	$(n{-}2100){+}(n{-}2002)$	$(n{-}2001)$
$(n000)$	$(n{-}1010)$	$(n{-}2100)$	$(n{-}2001)$	$(n{-}2000)$

Table 3: Spectrum of IIA supergravity on S^7, the multiplet $(\mathbf{n000})_{\rm IIA}$.

generators act vertically in this table, increasing the value of $\bar{\ell}_0$ from top to bottom by 1/2 per row. The value of ℓ_0 is increased from left to right by 1/2 per column which may be thought of as the action of the broken supersymmetry generators. The omission of any representation with negative Dynkin labels will be always understood. The full spectrum in table 2 may then be written as

$$\mathcal{H}_{\rm IIA} = \sum_{n=0}^{\infty} (\mathbf{n000})_{\rm IIA} = \sum_{n=0}^{\infty} (\mathbf{8_v} - \mathbf{8_s})(\mathbf{8_v} - \mathbf{8_s})(n000) \tag{3.11}$$

with products of SO(8) representations always understood as tensor products. The non-generic multiplicities in table 2 for small n precisely match with (3.11), taking into account that four-dimensional massless vector and spin-2 fields give rise to only two physical degrees of freedom in (3.9), (3.10) due to the additional gauge freedom in (3.3), (3.4). In particular, for $n = 0$ the physical degrees of freedom are entirely contained in first two columns of table 3; $n = 0$ states in the remaining three columns correspond to pure gauge degrees of freedom. The restriction to the first column at $n = 0$ has been argued to be a consistent truncation of supergravity on S^7 [20].

Alternatively, these results can be derived by decomposing short multiplets of the AdS_4 supergroup $\mathrm{OSp}(8|4;R)$ [21] in terms of supermultiplets of its $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{OSp}(8|2;\mathbb{R})_R$ subgroup [22], which is the conformal isometry of the background. The $\mathrm{ISO}(1,1) \times \mathrm{SO}(8)$ isometry group is instead generated by $L_{-1}, \bar{L}_{-1}, s_0 = L_0 - \bar{L}_0$ and S^7 Killing isometries. As we explain in more detail in the appendix, the columns of table 3 in fact correspond to primary subsets of the long multiplets of $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{OSp}(8|2;\mathbb{R})_R$, cf. tables 6–10. The E_0 quantum numbers refer to the eigenvalues of $L_0 + \bar{L}_0 = z\,\partial_z$.

In order to facilitate the comparison with the spectra of the other ten-dimensional supergravities, it is convenient to rewrite the spectrum in terms of $\mathcal{N} = (8,0)$ supermultiplets defined through

$$(\mathbf{n_1 n_2 n_3 n_4})_{\mathbf{s}} \equiv (\mathbf{8_v} - \mathbf{8_s})(n_1 n_2 n_3 n_4)\,. \tag{3.12}$$

In terms of these multiplets, the sum (3.11) read

$$\mathcal{H}_{\rm IIA} = \sum_{n=0}^{\infty} \left[(\mathbf{n+2000})_{\mathbf{s}} + (\mathbf{n+1001})_{\mathbf{s}} + (\mathbf{n100})_{\mathbf{s}} + (\mathbf{n010})_{\mathbf{s}} + (\mathbf{n000})_{\mathbf{s}} \right], \tag{3.13}$$

with every $\mathcal{N} = (8,0)$ multiplet spanning a column in table 3.

$(n{+}2000)$	$(n{+}1001)$	$(n100)$	$(n010)$	$(n000)$
$(n{+}1010)$	$(n{+}1000){+}(n011)$	$(n001){+}(n{-}1110)$	$(n{-}1100){+}(n{-}1020)$	$(n{-}1010)$
$(n100)$	$(n010){+}(n{-}1101)$	$(n000){+}(n{-}1011){+}(n{-}2200)$	$(n{-}1001){+}(n{-}2110)$	$(n{-}2100)$
$(n001)$	$(n-1100){+}(n{-}1002)$	$(n{-}1010){+}(n{-}2101)$	$(n{-}1000){+}(n{-}2011)$	$(n{-}2001)$
$(n000)$	$(n{-}1001)$	$(n{-}2100)$	$(n{-}2010)$	$(n{-}2000)$

Table 4: Spectrum of IIB supergravity on S^7, the multiplet $(\mathbf{n000})_{\rm IIB}$.

4. Type IIB supergravity on warped $AdS_3 \times S^7$

The KK harmonic analysis for type IIB on warped $AdS_3 \times S^7$ closely follows the one for the type IIA case above with some important differences. After reduction to SO(7), the type IIB field content $(\mathbf{8_v}+\mathbf{8_s})\times(\mathbf{8_v}+\mathbf{8_s})$ again gives rise to (3.1) and therefore to the same tower of SO(8) states in table 2. The structure of the supermultiplets however is substantially different. In contrast with the type IIA result (3.11), we expect for type IIB a decomposition according to

$$\mathcal{H}_{\rm IIB} = \sum_{n=0}^{\infty} (\mathbf{n000})_{\rm IIB} = \sum_{n=0}^{\infty} (\mathbf{8_v}-\mathbf{8_s})(\mathbf{8_v}-\mathbf{8_s})(n000)\,, \tag{4.1}$$

i.e. with a different assignment of quantum numbers $(\ell_0, \bar{\ell}_0)$ according to the flip of chirality in (4.1). These numbers may be extracted from table 4, where again the value of $\bar{\ell}_0$ increases from top to bottom by 1/2 per row whereas the value of ℓ_0 is increased from left to right by 1/2 per column. It is related to table 3 by interchanging the second and fourth column and simultaneous shift of n in these two columns.

Note that the two infinite sums (3.11) and (4.1) coincide and agree with table 2, although they correspond to different groupings of the fields into supermultiplets. The supermultiplets $(\mathbf{n000})_{\rm IIA}$ and $(\mathbf{n000})_{\rm IIB}$ share the first, third and fifth columns in tables 3 and 4. Bosonic modes in these columns are associated to the NSNS sector common to all five fundamental string theories. The associated states can be directly read off from the type IIA results in the previous section.

In the rest of this section, we shall confirm this structure of the IIB spectrum, by explicitly computing the linearized field equations around the type IIB string background. Similar KK techniques have been applied in [23] to the study of supergravity harmonics on $AdS_3 \times S^3$. We will mainly concentrate on the states corresponding to the second and fourth column of table 4 which cannot be derived from the IIA results of the last section, cf. appendix A, tables 7, 9. For definiteness, we will specify the analysis in this section to the D-string case. The spectrum associated to the supergravity harmonics around the fundamental string background is clearly the same, while NSNS and RR two-forms get exchanged under S-duality.

4.1 Equations of motion and background

The bosonic field equations for type IIB supergravity can be written as [24]

$$D^M P_M = \frac{1}{24} G_{MNP} G^{MNP}$$

$$
\begin{aligned}
D^P G_{MNP} &= -P^P G^*_{MNP} - \frac{2}{3}\, i\, F_{MNPQR}\, G^{PQR} \\
-R_{MN} &= P_M P^*_N + P^*_M P_N + \frac{1}{6}\, F_{MP_1\ldots P_4}\, F_N^{P_1\ldots P_4} \times \\
&\quad \times \frac{1}{8}\left(G_M{}^{PQ} G^*_{NPQ} + G^*_M{}^{PQ} G_{NPQ} - \frac{1}{6} g_{MN} G^{PQR} G^*_{PQR} \right) \\
F_{M_1\ldots M_5} &= \frac{1}{5!}\, \omega_{M_1\ldots M_5 N_1\ldots N_5}\, F^{N_1\ldots N_5}\,,
\end{aligned} \tag{4.2}
$$

with field strengths

$$
\begin{aligned}
G_{MNP} &= -\epsilon_{\alpha\beta}\, V^\alpha{}_+\, F^\beta_{MNP}\,, \\
F_{MNPQR} &= 5\, \partial_{[M}\, A_{NPQR]} + \frac{5}{8}\, i\epsilon_{\alpha\beta}\, A^\alpha_{[MN}\, F^\beta_{PQR]}\,,
\end{aligned} \tag{4.3}
$$

and $F^1 = F^{2\,*}$. Here and in the following, we denote by $\omega_{M_1\ldots M_D} = e\,\epsilon_{M_1\ldots M_D}$ the D-dimensional volume form. The dilaton-axion system ϕ, C_0 is encoded in the SU$(1,1)$ matrix

$$
\begin{aligned}
V^\alpha{}_\pm &= \frac{1}{2\sqrt{\tau_2}} \begin{pmatrix} i\bar\tau - 1 & i\tau - 1 \\ i\bar\tau + 1 & i\tau + 1 \end{pmatrix}, \qquad && \tau = \tau_1 + i\,\tau_2 = C_0 + ie^{-\phi}\,, \\
P_M &= \frac{i\partial_M \tau}{2\tau_2}\,, && Q_M = -\frac{\partial_M \tau_1}{2\tau_2}\,.
\end{aligned} \tag{4.4}
$$

The D-string background (2.26) in these variables is given by

$$
\mathring{\tau} = iz^{-3/2}\,, \qquad \mathring{P}_z = \frac{3}{4} z^{-1}\,, \qquad \mathring{G}_{+-z} = 3\, z^{-13/4}\, \ell^2\,, \tag{4.5}
$$

and metric (2.26). It preserves half of the original supersymmetry. We will expand the ten-dimensional fluctuations in terms of sphere harmonics $Y^\ell_{\mathbf{m}}(y)$

$$
\Phi_{\mu\mathbf{m}} = \sum_\ell \phi_{\mu(x)}\, Y^\ell_{\mathbf{m}}(y) \tag{4.6}
$$

with collective indices $\mu, \mathbf{m}$ carrying the SO(3) $\times$ SO(7) Lorentz representation of a given field. The sum over ℓ run over all possible SO(8) representations entering in the harmonic decomposition, cf. table 1 and will typically be described by an integer k. For clearness, we will in general suppress the indication of the explicit dependence of $\phi(x)$ on x^μ and $Y^\ell_{\mathbf{m}}(y)$ on y^m, as well as the index ℓ, whenever non-ambiguous.

4.2 Three-dimensional equations

After reduction on S^7, the bosonic ten-dimensional equations (4.2) give rises to a set of equations associated to particles of spin 0, 1, 2 moving on warped AdS_3. The arising scalar field equations will be of the general form (2.7). As we have discussed above, this form of equation is compatible with a rescaling (2.13) under which the energy of the field changes — in agreement with the fact that the energy is associated with $L_0 + \bar{L}_0 = z\partial_z$ which is not a Killing vector of the background. In order to allow a comparison between the different scalar equations, we will transform all of them into the form of the modified second order equation on pure AdS space (3.5), which naturally appeared in the type IIA reduction, cf. also (2.7), (2.8).

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Likewise, we shall bring the arising vector equations into the form (3.7). This is accomplished by noting that the equation

$$D^\mu\left(z^\gamma F_{\mu\nu}\right)=\frac{1}{\tilde{\ell}^2}\,\lambda\, z^\gamma A_\nu\,. \tag{4.7}$$

on pure AdS_3 may be transformed into (3.7) by means of the rescaling

$$A_\mu\to z^{\frac{1}{2}(1+\gamma)}\left(A_\mu+\frac{1+\gamma}{p^2 z}\left(\partial_\mu A_z-A'_z\delta^z_\mu\right)\right),\qquad \lambda\to\lambda-1+\gamma^2\,. \tag{4.8}$$

In addition, we shall encounter another type of vector field equations in the IIB reduction, namely the first order equations

$$F_{\mu\nu}=m\,\omega_{\mu\nu\rho}\,A^\rho\,, \tag{4.9}$$

on pure AdS_3. Iterating this equation leads to (4.7) with $\gamma=0$, i.e. upon rescaling according to (4.8) it induces the standard form (3.7) with

$$E_0=\frac{3}{2}+\frac{1}{2}|m|\,. \tag{4.10}$$

Note that unlike (3.7), equation (4.9) gives rise to only one physical degree of freedom.

4.3 Scalar sphere harmonics

Let us first consider fluctuations containing the scalar sphere harmonics $Y^{(k)}$, associated with the representation ($k000$) and satisfying

$$\Box_{S^7}Y^{(k)}=-\frac{1}{4}\,k(k+6)\,Y^{(k)}\,. \tag{4.11}$$

The extra factor of $1/4$ on the r.h.s. comes from the S^7 radius $\tilde{\ell}^2=4\,\ell^2$ while ℓ^2 will be set to one throughout this section. The scalar sphere harmonics $Y^{(k)}$ appear in the expansion of the following fluctuations

$$\begin{aligned}
g_{\mu\nu}&=\mathring{g}_{\mu\nu}\left(1+h_3\,Y^{(k)}\right)+h_{\mu\nu}\,Y^{(k)}\,,\qquad \mathring{g}^{\mu\nu}h_{\mu\nu}=0\,,\\
g_{mn}&=\mathring{g}_{mn}\left(1+h_7\,Y^{(k)}\right),\\
\tau&=ie^{-\mathring{\phi}}-ie^{-\mathring{\phi}}\left(i\chi+\varphi\right)Y^{(k)}\,,\\
A_{\mu\nu}&=\mathring{A}_{\mu\nu}+\left(b_{\mu\nu}+i\,c_{\mu\nu}\right)Y^{(k)}\,,
\end{aligned} \tag{4.12}$$

The Einstein field equations imply $3h_3+5h_7=0$ which we shall use to eliminate h_3. The $[\mu m]$ component of the three-form equations of motion imply the following reduction

$$b_{\mu\nu}=z^{-1}\omega_{\mu\nu\rho}\,\partial^\rho b\,,\qquad c_{\mu\nu}=z^2\omega_{\mu\nu\rho}\,\partial^\rho\left(z^{-3/2}c\right) \tag{4.13}$$

The rescaling of c turns out to be convenient in the following. The fluctuation equations for χ and c decouple from the others and after some computation give rise to

$$\begin{aligned}
z^2\chi_1''-2z\chi_1'+\frac{1}{4}\left(9-k^2\right)\chi_1+p^2z^2\chi_1&=0\,,\\
z^2\chi_2''-2z\chi_2'-\frac{1}{4}\left(27+12k+k^2\right)\chi_2+p^2z^2\chi_2&=0\,,
\end{aligned} \tag{4.14}$$

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for the combinations

$$\chi_1 = \chi + \frac{1}{6}(6+k)\,c\,, \qquad \chi_2 = \chi - \frac{1}{6}k\,c\,. \tag{4.15}$$

These equations are indeed of the scalar type (3.5) with energies

$$E_0^{(1)} = \frac{1}{2}k + \frac{3}{2}\,, \qquad E_0^{(2)} = \frac{1}{2}k + \frac{9}{2}\,. \tag{4.16}$$

Comparing to table 4 we hence find precise agreement with the scalars in the second and fourth column — note that $k = n \pm 1$. The remaining three scalar fields (φ, b, h_7) are somewhat more difficult to analyze, as they mix with the fluctuations of the three-dimensional metric. It turns out, that the combinations

$$\phi_1 = 6\,h_7 + \varphi - \frac{1}{3}\,(6+k)\,b\,, \qquad \phi_2 = 6\,h_7 + \varphi + \frac{1}{3}\,k\,b\,, \tag{4.17}$$

satisfy separate equations of motion

$$\begin{aligned} z^2\phi_1'' - 2z\phi_1' + \frac{1}{4}k(6-k)\,\phi_1 + p^2z^2\phi_1 &= 0\,, \\ z^2\phi_2'' - 2z\phi_2' - \frac{1}{4}(72+18k+k^2)\,\phi_2 + p^2z^2\phi_2 &= 0\,, \end{aligned} \tag{4.18}$$

which again are equations of the type (3.5) with energies

$$E_0^{(1)} = \frac{1}{2}k\,, \qquad E_0^{(2)} = \frac{1}{2}k + 6\,. \tag{4.19}$$

This corresponds to the scalars in tables 6 and 10, i.e. the first and last column of table 4 ($k = n \pm 2$). The fifth scalar in this sector is more complicated to identify. At momentum $\mathbf{p} = 0$, we find that the combination

$$\phi_3 = 6(16 + k(k{+}6))\,h_7 - 3(6 + k(k{+}6))\,\varphi + k(k{+}6)\,b + 2z\left(18h_7' + 3\varphi' - 4b'\right),$$

satisfies scalar field equations (3.5) with energy $\frac{1}{2}k{+}3$ in agreement with the third column of table 4, cf. table 8.

4.4 Vector harmonics

We consider now fluctuations carrying an SO(7) vector index, i.e. the scalar sphere harmonics $Y_m^{(k)}$ associated with the representation $(k-1100)$ and satisfying

$$\Box_{S^7}\,Y_m^{(k)} = -\frac{1}{4}\,[k(k+6)-1]\;Y_m^{(k)}\,, \qquad D^m\,Y_m^{(k)} = 0\,. \tag{4.20}$$

They appear in the field fluctuations

$$\begin{aligned} A_{\mu\nu\sigma m} &= c_{\mu\nu\sigma}\,Y_m^{(k)}\,, \\ A_{\mu m} &= (b_\mu + i\,c_\mu)\,Y_m^{(k)}\,, \\ h_{\mu m} &= h_\mu\,Y_m^{(k)}\,. \end{aligned} \tag{4.21}$$

The self-duality equations for the five-form

$$F_{\mu\nu\sigma m_1 m_2} = 2\,\partial_{[m_1}\,Y_{m_2]}\,\left(c_{\mu\nu\sigma} + \frac{3}{8}\,\mathring{A}_{[\mu\nu}\,c_{\sigma]}\right) = 0\,,$$

can be used to determine $c_{\mu\nu\sigma}$ in terms of c_σ. The remaining equations split as before into real and imaginary parts. We focus on the fields in the second and fourth column of table 4, i.e. those coming from the imaginary part. By plugging the ansatz (4.21) into the three-form equation in (4.2), one finds

$$\hat{D}_P\,\left(z^{-3/4}\,\hat{F}^{P\mu n}\right) - z^{-3/4}\,\mathring{P}_z\,\hat{F}^{z\mu n} = 0$$

with

$$F_{m\mu n} = 2\,\partial_{[n}Y_{m]}\,c_\mu\,, \qquad F_{\nu\mu n} = 2\,\partial_{[\nu}\,c_{\mu]}\,Y^n\,.$$

Using (4.5), this equation may be rewritten in pure AdS_3 background as

$$D^\nu\,\left(z^{-2}\,F_{\nu\mu}\right)\,Y^n + z^{-2}\,c_\mu\,\left(\Box_{S^7} - \frac{3}{2}\right)\,Y^n = 0\,, \tag{4.22}$$

which reduces to a vector equation of type (4.7) with $\gamma = -2$, $\lambda = \frac{1}{4}(k^2 + 6\,k + 5)$. Upon rescaling according to (4.8), this gives rise to the standard vector equation (3.7) with energy

$$E_0 = \frac{1}{2}k + 3\,. \tag{4.23}$$

This reproduces the values in tables 7 and 9 ($k = n$) in this sector. A similar analysis of the real part of the fluctuation equations corresponds to the vector fields in the odd columns of table 4 and yields the energies

$$E_0^{(1)} = \frac{1}{2}k + \frac{3}{2}\,, \qquad E_0^{(2)} = \frac{1}{2}k + \frac{9}{2}\,, \tag{4.24}$$

with $k = n \pm 1$. In particular, for the SO(8) gauge vector fields arising at $n = 0$, this gives $\delta = 1/2$, cf. (3.6). Computing the associated current-current correlation functions according to (2.25), we hence recover the result of [5].

4.5 Two-form harmonics

Next, we analyze the fluctuations involving the rank two form $Y^{(k)}_{mn}$ on S^7 associated to the representation $(k - 1011)$:

$$3\,D^m\,D_{[m}\,Y^{(k)}_{np]} = -\frac{1}{4}(k^2 + 6k + 8)\,Y^{(k)}_{np} \equiv \lambda\,Y^{(k)}_{np}\,, \qquad D^m Y^{(k)}_{mn} = 0\,. \tag{4.25}$$

They appear as

$$\begin{aligned} A_{mn} &= (b + i\,c_1)\,Y_{mn}\,, \\ A_{\mu\nu mn} &= c_{\mu\nu}\,Y_{mn}\,, \\ A_{m_1\ldots m_4} &= \frac{c_2}{3!}\,\hat{\omega}_{m_1\ldots m_4}{}^{m_5\ldots m_7}\,\partial_{m_5}\,Y_{m_6 m_7}\,. \end{aligned} \tag{4.26}$$

Plugging (4.26) into the five-form self-duality equations we find

$$\begin{aligned} \frac{1}{3}\,\lambda\, c_2\, \omega_{\mu\nu\rho} &= 3\,\partial_{[\rho}\, C_{\mu\nu]} - \frac{1}{4}\mathring{H}_{\mu\nu\rho}\, c_1\,, \\ C_{\mu\nu} &= -\frac{1}{3}\,\omega_{\mu\nu\rho}\partial^\rho\, c_2\,, \end{aligned} \tag{4.27}$$

with $C_{\mu\nu} \equiv c_{\mu\nu} + \frac{1}{8}\mathring{A}_{\mu\nu}\, c_1$, and λ defined through (4.25). The imaginary part of the three-form equation

$$\hat{D}^P\, G_{mnP} + \mathring{P}^z\, G^*_{mnz} = -\frac{2}{9}\, i\,\lambda\, c_2\, \hat{\omega}^{\mu\nu\rho}\, \mathring{H}_{\mu\nu\rho}\, Y_{mn}\,,$$

reduces to

$$z^5\,\partial_\mu\left(z^{-3}\partial^\mu\, c_1\right) + \lambda\, c_1 - 4\,\lambda z\, c_2 = 0\,. \tag{4.28}$$

Equation (4.27) can be used to solve for $C_{\mu\nu}$ in terms of c_2. Combining with (4.28), we are left with the following system of second order differential equations:

$$\begin{aligned} z^2\,\bar{c_2}'' - 2\,z\,\bar{c_2}' + p^2\,z^2\,\bar{c_2} - \frac{1}{4}(k(k{+}6) + 3)\,\bar{c_2} + \frac{9}{4}\,\bar{c_1} &= 0\,, \\ z^2\,\bar{c_1}'' - 2\,z\,\bar{c_1}' + p^2\,z^2\,\bar{c_1} - \frac{1}{4}(k(k{+}6) + 15)\,\bar{c_1} + (k(k{+}6) + 8)\,\bar{c_2} &= 0\,, \end{aligned} \tag{4.29}$$

with $\bar{c_1} = z^{-\frac{1}{2}}\, c_1$, $\bar{c_2} = z^{\frac{1}{2}}\, c_2$. This rescaling is required to bring the equations into the standard form (3.5). After straightforward diagonalization they give rise to differential equations of the standard type (2.7), (2.8) with energies

$$E_0^- = \frac{1}{2}k + \frac{3}{2}\,, \qquad E_0^+ = \frac{1}{2}k + \frac{9}{2}\,. \tag{4.30}$$

These energies match the values in tables 7, 9 ($k = n \pm 1$).

4.6 Three-form harmonics

Finally, we consider fluctuations of the self-dual five form carrying the rank three sphere harmonics $Y^{(k\pm)}_{mnp}$ satisfying

$$\frac{1}{3!}\,\omega_{m_1\ldots m_7}\, D^{m_1}\, Y^{(k\pm)\,m_2m_3m_4} = \pm\frac{1}{2}(k+3)\, Y^{(k\pm)}_{m_5m_6m_7}\,, \tag{4.31}$$

with the sign $\pm$ distinguishing the two representations $(k-1020)$ and $(k-1002)$.

Inspection of table 4 suggests that these fluctuations give rise to the vector fields appearing in the second and fourth column. Recall that the vector fields (4.21) satisfy the standard equation (3.7) giving rise to two physical degrees of freedom. In contrast, the representations $(k-1020)$, $(k-1002)$ each appear only once in this table. This already suggests that these fields rather satisfy a first order equation which indeed will turn out to be the case. Consider the ansatz

$$A_{\mu mnp} = c_\mu\, Y^{(k\pm)}_{mnp}\,, \qquad A_{mnpq} = c\, D_{[m} Y^{(k\pm)}_{npq]}\,. \tag{4.32}$$

$(n{+}2000)$	$(n100)$	$(n000)$	$n_v \cdot (n{+}1000)$
$(n{+}1010)$	$(n001){+}(n{-}1110)$	$(n{-}1010)$	$n_v \cdot (n010)$
$(n100)$	$(n000){+}(n{-}1011){+}(n{-}2200)$	$(n{-}2100)$	$n_v \cdot (n{-}1100)$
$(n001)$	$(n{-}1010){+}(n{-}2101)$	$(n{-}2001)$	$n_v \cdot (n{-}1001)$
$(n000)$	$(n{-}2100)$	$(n{-}2000)$	$n_v \cdot (n{-}1000)$

Table 5: Spectrum of type I supergravity on S^7.

The $[m_1 \dots m_5]$ component of the five-form self-duality equations implies $c = 0$, while the $[\mu\nu m_1 m_2 m_3]$ component reduces to the form (4.9)

$$2\,\partial_{[\mu}\,c_{\nu]}\,Y^{(k\pm)}_{mnp} = \pm\frac{1}{2}(k+3)\,\omega_{\mu\nu\rho}\,c^\rho\,Y^{(k\pm)}_{mnp}\,. \tag{4.33}$$

According to our discussion above, this gives rise to fields with energy

$$E_0^\pm = \frac{1}{2}k + 3\,, \tag{4.34}$$

in agreement with the values in tables 7, 9 ($k = n$).

5. Type I supergravity on warped $AdS_3 \times S^7$

In this section, we consider the reduction of type I supergravity on warped $AdS_3 \times S^7$. The starting point is now given by the decomposition of the ten-dimensional $(\mathbf{8_v}{+}\mathbf{8_s})(\mathbf{8_v}{+}n_v{\cdot}\mathbf{1})$ (with n_v the number of vector multiplets) field content under SO(7):

$$R^{\rm I}_{\rm SO(7)} = (2{+}n_v)\cdot\mathbf{1} + (2{+}n_v)\cdot\mathbf{7} + (2{+}n_v)\cdot\mathbf{8} + \mathbf{21} + \mathbf{27} + \mathbf{48}\,. \tag{5.1}$$

Combining this with table 1 one finds:

$$\mathcal{H}_I = \sum_{n=0}^{\infty}(\mathbf{8_v} + n_v\cdot\mathbf{1})(\mathbf{n000})_s\,, \tag{5.2}$$

with the multiplet $(\mathbf{n000})_s$ given in (3.12) above. The entire spectrum is collected in table 5. The quantum numbers for states sitting in the first three columns can be read off directly from those in type IIB computed above. In the rest of this section we shall hence analyze the remaining last column which collects the states coming from the gauge sector of type I.

The linearized bosonic equations in the type I gauge sector read

$$\hat{D}_M\,(e^{\mathring{\phi}/2}\,\hat{F}^{MN}) - \frac{1}{2}\,e^{\mathring{\phi}}\,\mathring{H}_{MNP}\,\hat{F}^{NP} = 0\,, \tag{5.3}$$

with $F_{MN} = 2\,\partial_{[M}\,A_{N]}$. Plugging in the vacuum solution (2.26) one finds

$$\begin{aligned} D_N\,F^{Nm} &= \left(\Box_{AdS} + \Box_{S^7} - \frac{3}{2}\right)\,A^m = 0\,, \\ D_N\,F^{N\mu} - \frac{3}{2}\,\omega^{\mu\nu\rho}\,F_{\nu\rho} &= D_\nu\,F^{\nu\mu} + \Box_{S^7}\,A^\mu - \frac{3}{2}\,\omega^{\mu\nu\rho}\,F_{\nu\rho} = 0\,. \end{aligned} \tag{5.4}$$

Remarkably, all the explicit z-dependence in these equations has been canceled against the non-trivial dependence of the dilaton, three form and warped metric. The gauge theory effectively lives on pure $AdS_3 \times S^7$. Consistently, the type I gauge multiplet (the last column of table 5, cf. table 11) matches the structure of short multiplets of $\mathrm{OSp}(8|,2,\mathbb{R})_R \times \mathrm{SL}(2,\mathbb{R})_L$ found in [22]. We will hence in this sector bring all equations into the pure AdS form. Scalar fields for example will be normalized such as to satisfy (2.7), (2.8) with $a = 1$ rather than $a = 3/2$, etc.

The first equation in (5.4) for the three-dimensional scalars A_m can be solved in terms of the ansatz

$$A_m = a\, Y_m^{(k)}\,, \tag{5.5}$$

with $Y_m^{(k)}$ associated to the representation $(k-1100)$ and satisfying (4.20). Plugging this into (5.4) we are left with an equation of scalar type (2.7) with $a = 1$ and

$$E_0 = \frac{1}{2}k + \frac{5}{2}\,, \tag{5.6}$$

which matches the result for the scalar component in table 11. The equations of motion for the three-dimensional vector components on the other hand can be solved by means of the ansatz

$$A_\mu = e_\mu^a\, a_a\, Y^{(k)}\,. \tag{5.7}$$

They reduce to two sets of first order vector equations (4.9) with

$$m_\pm = \frac{3}{2} \pm \frac{1}{2}(k+3)\,. \tag{5.8}$$

Energies can be read off from (4.10) and are given by

$$E_0^- = \frac{1}{2}k + 1\,, \qquad E_0^+ = \frac{1}{2}k + 4\,. \tag{5.9}$$

Recalling that $Y^{(k)}$ is associated to the representation $(k000)$ we see that these energies match the values quoted in table 11.

6. Chiral primaries in the gauge theories

In the previous sections we have determined the spectrum of masses and charges of single particle Kaluza-Klein states in the reduction of ten-dimensional supergravities on warped $AdS_3 \times S^7$. The aim of this section is to compare these results to their dual descriptions in terms of primary operators in the boundary theories.

The nature of the dual theory is substantially different depending on whether we consider systems of fundamental or D-strings. In the former case, states in a floor of the Kaluza Klein tower below level N are associated to fundamental string states with charge N. The physics at finite g_s is presumably described by deformations of the two-dimensional $\mathcal{N} = (8,8)$ and $\mathcal{N} = (8,0)$ sigma models associated to ten-dimensional strings on flat spacetimes. Systems involving fundamental strings are particularly interesting since due to the absence of RR backgrounds string theory can be handled in a better controlled way. In the D-string case the dual gauge theory is defined via quantization of the lowest open string modes governing the low energy dynamics of N nearby D-strings. They result into effective U(N) and SO(N) two-dimensional gauge theories for the type II and type I D-string, respectively. We focus our study on these D-string systems.

Let us start by considering the D-string system in type IIB. The effective U(N) gauge theory describing the low energy dynamics of N nearby D-strings is obtained from dimensional reduction of $\mathcal{N} = 1$ SYM in $D = 10$ down to two dimensions. The field content comprises (besides the gauge vector field) eight adjoint scalars ϕ^I, and left and right moving fermions S^a, $S^{\dot{a}}$, transforming in the $\mathbf{8_v}$, $\mathbf{8_s}$, and $\mathbf{8_c}$, respectively, of the SO(8) R-symmetry group. The theory is manifestly invariant under sixteen supersymmetries. The Poincaré symmetry group ISO(1, 1) $\times$ SO(8) and supersymmetries match the isometries and Killing spinors of the $AdS_3 \times S^7$ background (2.26), as expected.

The analysis of primary operators follows straightforwardly from that of $\mathcal{N} = 4$ SYM from which the two-dimensional gauge theory descends upon dimensional reduction. As in the four-dimensional case, chiral primaries are associated to completely symmetrized, traceless operators $\mathcal{O}_m \equiv \mathrm{Tr}\left(\phi^{I_1} \ldots \phi^{I_m}\right)$, $m = 2, 3, \ldots$ built from scalar fields in the $\mathbf{8_v}$ and transforming in the (m000) representation of the SO(8) R-symmetry group. The missing of the $m = 1$ state is due the fact that ϕ^I is an SU(N) rather than a U(N) matrix. The remaining primaries can be found by acting with the fermionic charges $Q^a, \widetilde{Q}^{\dot{a}}$ on the chiral primary. Half of these charges Q^a realize the eight on-shell supersymmetries and span the columns of table 4. The remaining $\widetilde{Q}^{\dot{a}}$ act horizontally between the various columns. The state at the bottom right is reached by $Q^4\, \widetilde{Q}^4 \mathcal{O}_m = \mathrm{Tr}\left(\mathcal{F}^4\, \phi^{I_1} \ldots \phi^{I_{m-4}}\right)$. The resulting spectrum of SO(8) representations and spins is listed in tables 6–10 in the appendix ($m = n{+}2$). The results agree with those coming from the KK harmonic analysis in the last section. The E_0 quantum number can also be related to conformal dimensions in the parent SYM in $D = 4$. Recalling that $E_0 = \frac{1}{2}\Delta$, we see that the chiral primary $\mathcal{O}_m$ carries a total energy $E_0 = \frac{1}{2}m$ in agreement with the supergravity result. Notice that this result generalizes straightforwardly to arbitrary dimensions: chiral primaries at the floor m in the KK reductions on $AdS_{d+1} \times S^{9-d}$ will carry energies $E_0 = \frac{\ell}{\ell} m = \frac{(6-d)}{2} m$ in agreement with (2.11).

It is interesting to compare the above results with the $\mathcal{N} = (8, 8)$ SCFT describing the infrared physics. Although the supergravity picture breaks down in this regime, one can still hope that protected quantities like the spectrum of chiral primaries, and their two- or three-point functions are preserved under the flow towards the IR. According to [25], chiral primaries in a symmetric product SCFT M^N/S_N are counted by an elliptic index which can be thought as a second quantized partition function for the non-trivial cohomologies of M. In our case, this leads to a single generator associated to $h_{0,0} = 1$ in $\mathbb{R}^8$. This is in agreement with the results in the supergravity side where a single "$\mathcal{N} = (8, 8)$ chiral primary"[1] was found at each level m in the KK tower. Despite this encouraging evidence of a correspondence between the spectra of SCFT/supergravity chiral primaries, the mismatch between the supergroups in the SCFT and supergravity regimes still obscures the precise

[1]By this, we mean a state at the top left in table 4. In the conformal limit all states in this table should come together to build a multiplet of the maximal $\mathcal{N} = (8, 8)$ supersymmetry.

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dictionary between masses and charges. It would be nice to perform a systematic study of the SCFT/supergravity elliptic genera along the lines of [19, 26] to put this correspondence on more firm grounds.

Finally, let us comment on the $n = -1$ states in table 4. These states, missing in the gauge/supergravity spectrum are the so called singletons and typically associated with global degrees of freedom living at the AdS boundary. In the present situation, they include $\mathbf{8}_v$ scalars and $\mathbf{8}_s, \mathbf{8}_c$, spin-1/2 fields which match the worldsheet content of the dual IR CFT. In addition, there is an extra scalar at $n = -1$ in table 7 which transforms as a singlet under SO(8). This state corresponds to a rather special operator in the dual gauge theory. From table 4 we see that this mode is associated to the first irrelevant operator invariant under the whole $\mathcal{N} = (8, 8)$ supersymmetry. These are precisely the requisites met by the DVV $\mathbb{Z}_2$-twisted operator [6] associated to deformations of the IR SCFT away from the $(\mathbb{R}^8)^N/S_N$ orbifold point. Its dimension $\Delta = 2E_0 = 3$ and spin $s_0 = 0$ also match those of the DVV operator in [6].

The above analysis can be easily extended to the type I case. The D-string bound state dynamics is now described by a $O(N)$ gauge theory with eight scalars ϕ^I and right moving fermions S^a in the symmetric representation of $O(N)$, eight left moving adjoint fermions $S^{\dot{a}}$, and 32 left moving λ^i fermions transforming as singlets of SO(8) and bifundamentals of $O(N) \times \mathrm{SO}(32)$. The different $O(N)$ representations of left and right moving fermions compensate for the relative sign in the action of the Ω-projection on type IIB D-string fields. The surviving supersymmetry is $\mathcal{N} = (8, 0)$. The extra fundamentals arise from quantization of D1-D9 open strings. Chiral primaries associated to bulk supergravity modes can be derived from those in type IIB theory via Ω-projection. In order to do this we first decompose the $\mathcal{N} = (8, 8)$ supermultiplets of table 4 in terms of $\mathcal{N} = (8, 0)$ as given by tables 6–10, collected in the appendix; each of them has an $\mathcal{N} = (8, 0)$ chiral primary on the top. Operators sitting in tables 6, 8, 10, are kept by the Ω-projection and the SO(8), spin and scaling quantum numbers follow from their type IIB cousins. From the gauge theory point of view, the projection of operators in the second and fourth columns of table 4 can be easily understood since they are associated to traces involving an odd number of antisymmetric matrices. In addition, we have $n_v = \dim[\mathrm{SO}(32)]$ extra $\mathcal{N} = (8, 0)$ multiplets descending from the gauge chiral primary Tr $\left(\lambda^{[i}\,\lambda^{j]}\,\phi^{I_1}\ldots\phi^{I_{m-1}}\right)$ in the $(m-1000)$ representation of SO(8). This agree with the supergravity spectrum in table 11. The energies E_0 also match the values in the table with an extra contribution of 1 in the gauge sector coming from $\lambda^{[i}\,\lambda^{j]}$.

In the infrared limit the gauge theory flows to a two-dimensional SCFT given in terms of N copies of heterotic strings moving on $(\mathbb{R}^8)^N/S_N$ [9]. The singletons at $n = -1$ now include, besides the $\mathbf{8_v}$ scalars and $\mathbf{8_s}$ right moving fermions, the SO(8) singlets $\lambda_L^{[i}\,\lambda_L^{j]}$ in the adjoint of the SO(32) D9-gauge group in agreement, with the SCFT worldsheet content. The spectrum of $\mathcal{N} = (8, 0)$ chiral primaries in the IR SCFT is now associated to a second quantized partition function with $n_v + 8$ bosonic generators, coming from chiral primaries at the massless level of a single copy of the heterotic string on $\mathbb{R}^8$. Upon compactifications on a circle the SO(32) heterotic gauge group is generically broken to $\mathrm{U}(1)^{16}$ giving $n_v = 16$. This agrees with the supergravity results above, with $n_v + 8$ the number of (single-particle) chiral primaries found at each level in the KK tower.

7. Conclusions

There are several interesting questions raised by our study. As has been remarked in [3], the vacuum configuration (2.26) implies the existence of gauged supergravities which do not admit a ground state but rather a domain wall solution. The field content of these theories is given by the supermultiplets $(\mathbf{0000})_{\mathrm{IIA}}$ and $(\mathbf{0000})_{\mathrm{IIB}}$, in tables 3 and 4, respectively. Whereas the former theory may just be obtained from the circle reduction of the SO(8) gauged four-dimensional theory of [27], the latter theory is as yet unknown. We suspect that both these theories may in fact be embedded into the maximal gauged three-dimensional supergravity constructed in [28] which admits an $\mathcal{N} = (8, 8)$ AdS_3 vacuum. It would be interesting to understand within this theory the flow of the matrix string models to the orbifold SCFT's with target space $(\mathbb{R}^8)^N/S_N$ and enhanced supersymmetries in the infrared limit. Domain wall solutions interpolating between two-dimensional SCFT's have been previously studied in [29].

In [30], the authors have shown that the string genus expansion is reproduced by a simple counting of moduli in the matrix string SCFT. It would be nice to understand the meaning of this result from the supergravity perspective.

The analysis of domain wall/QFT duals in other dimensions is of obvious interest. In [31], an exact CFT description of the infrared dynamics of D5-branes in type I theory was proposed in terms of second quantized type II strings on $(\mathbb{R}^4 \times K3)^N/S_N$. The study of this SCFT from the dual supergravity perspective could provide further insights in the physics content of these domain wall/QFT correspondences.

Another interesting direction is to use the $AdS_3 \times S^7$ dual description to explore the non-perturbative physics described by type II and heterotic matrix string models. Similarly, one could exploit the $AdS_2 \times S^8$ dual description of D0 matrix theories [32] in order to study M-theory physics. The celebrated matrix theory description of 11d graviton-graviton scattering in terms of $\mathrm{Tr}\,\mathcal{F}^4$ correlation function in SYM in this perspective translates into a tadpole computation of the corresponding dual supergravity field in the string background. For $d = 2$, the associated supergravity field corresponds to the state at the bottom-right of table 4 for $n = 2$. We have identified this field in (4.17) as a linear combination ϕ_2 of the s-waves ($k = 0$) of the dilaton and the metric trace on S^7. Remarkably, this particular linear combination does not involve the RR b-field. It is interesting to compare this with the results of [33], where the $\mathrm{Tr}\,\mathcal{F}^4$ term in SYM with 16 supercharges was shown to be reproduced by a tadpole computation again purely in terms of NSNS fields.[2] It would be nice to have more quantitative tests of these ideas.

Finally, it is natural to investigate the pp-wave limits [34] of the correspondences under study here. Penrose limits of near horizon Dp-brane geometries have been recently studied in [35]. We hope to come back to some of these issues in the near future.

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[2]We thank M. Bianchi for drawing our attention to this point

A. $\mathcal{N} = (8,0)$ short multiplets

In this appendix we display the $\mathrm{ISO}(1,1) \times \mathrm{SO}(8)$ quantum numbers and the structure of $\mathcal{N} = (8,0)$ supermultiplets entering in KK reductions of ten-dimensional supergravities on S^7. The $\mathcal{N} = (8,0)$ supersymmetry is chosen here as the basic unit common to all five ten-dimensional strings in order to present a unifying treatment. States in the reductions of type II strings are naturally organized in terms of "$\mathcal{N} = (8,8)$ supermultiplets" displayed in tables 3, 4 which decompose into sixteen $\mathcal{N} = (8,0)$ supermultiplets transforming as $\mathbf{8_v}+\mathbf{8_c}$ or $\mathbf{8_v}+\mathbf{8_s}$ of the SO(8) Lorentz group for type IIA and IIB, respectively. We denote by Q the supersymmetry charges acting inside of $\mathcal{N} = (8,0)$ supermultiplets. These corresponds to the eight on-shell supersymmetries. Fermionic charges bringing us from one $\mathcal{N} = (8,0)$ supermultiplet to another one inside a big "$\mathcal{N} = (8,8)$ multiplet" will be denoted by $\bar{Q}(\widetilde{Q})$ for type IIA (IIB).

From a two-dimensional point of view, supersymmetry charges (as two-dimensional fermions) transform naturally under $\mathrm{SO}(8)_L \times \mathrm{SO}(8)_R$ but only the diagonal SO(8) Lorentz group is preserved by gauge interactions. The displayed SO(8) quantum numbers will always refer to this diagonal subgroup. States will be labeled by the SO(8) Dynkin labels (n_1, n_2, n_3, n_4) and $(\ell_0, \bar{\ell}_0)$ quantum numbers. The eight supersymmetry charges Q decompose under $\mathrm{SO}(2) \times \mathrm{SO}(6) \sim \mathrm{U}(1)_F \times \mathrm{SU}(4)$ as four creation and four annihilation with $\mathrm{U}(1)_F$ charges $q = -1/2$ and $q = 1/2$, respectively. The same holds for $\bar{Q}, \widetilde{Q}$. The SO(8) content of a given state can be entirely determined from its $\mathrm{U}(1)_F \times \mathrm{SU}(4)$ quantum numbers via the dictionary [21]

$$[(a_1, a_2, a_3); q] \to \left(q + \frac{1}{2}(n_3 - n_1 - n_2), n_2 - n_3, n_1 - n_2, n_3 \right), \tag{A.1}$$

with (a_1, a_2, a_3) the numbers of boxes in each row of the Young tableaux diagram characterizing the SU(4) representation. In addition, fermionic charges Q raise by 1/2 the value of ℓ_0, while $\bar{Q}, \widetilde{Q}$ raise that of $\bar{\ell}_0$.

Let us start by considering the type IIA spectrum. We start from the highest weight state $|\,\Omega\,\rangle = |0\rangle$ with $\left[(n+2000); \frac{1}{4}(n+2), \frac{1}{4}(n+2)\right]$ quantum numbers. The states on the top of tables 6–10 are found by iterative actions of $\bar{Q}$ on $|\Omega\rangle$. States in the same table can be reached via Q supersymmetry actions. The two set of fermionic charges are in the fundamental of SU(4) and carry the following quantum numbers:

$$\begin{aligned} Q: \qquad & \left[(a_1, a_2, a_3), q; \ell_0, \bar{\ell}_0\right] = \left[(1,0,0), -\frac{1}{2}; 0, \frac{1}{2}\right] \\ \bar{Q}: \qquad & \left[(a_1, a_2, a_3), q; \ell_0, \bar{\ell}_0\right] = \left[(1,0,0), -\frac{1}{2}; \frac{1}{2}, 0\right] \end{aligned} \tag{A.2}$$

The remaining quantum numbers displayed in the tables are the scaling dimensions E_0 and spins s_0 related to the $(\ell_0, \bar{\ell}_0)$ through

$$E_0 = \ell_0 + \bar{\ell}_0 \,, \qquad s_0 = \ell_0 - \bar{\ell}_0 \,. \tag{A.3}$$

The type IIB spectrum follows in the same way with the only difference that the $\widetilde{Q}$ supercharges carry now opposite chirality, i.e.

$$\widetilde{Q}: \qquad [(a_1, a_2, a_3), q; \ell_0, \bar{\ell}_0] = \left[(1,1,1), -\frac{1}{2}; 0, \frac{1}{2}\right] \tag{A.4}$$

This gives rise to the SO(8) contents displayed in tables 7 and 9. Tables 6, 8, 10 coincide with those for type IIA since the highest weight state are now in real SO(8) representations. Finally, the spectrum of type I on S^7 can be obtained by keeping the multiplets in tables 6, 8, 10, and adding n_v gauge multiplets with content given by table 11.

It is instructive to understand these results from the representation theory of the group of conformal isometries $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{OSp}(8|2,\mathbb{R})_R$. Its unitary representations have been constructed in [22] on the Fock space of super-oscillators

$$\xi_R^A = \begin{pmatrix} \mathbf{a}^\dagger \\ \alpha^i \end{pmatrix}_R, \quad \eta_R^A = \begin{pmatrix} \mathbf{b}^\dagger \\ \beta^i \end{pmatrix}_R, \qquad \mathbf{a}_L^\dagger,\ \mathbf{b}_L^\dagger, \qquad i = 1, \ldots, 4, \tag{A.5}$$

with $p = m/2$ pairs of bosonic oscillators combined into vectors $\mathbf{a}_R$, $\mathbf{b}_R$, $\mathbf{a}_L$, $\mathbf{b}_L$ and similarly p pairs of fermionic oscillators α^i, β^i, together transforming in the fundamental representation of U(1|4) (for odd $m = 2p+1$ one includes an additional super-oscillator). The noncompact $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SL}(2,\mathbb{R})_R$ subgroup is realized in this formalism in terms of bilinears in the bosonic generators while the compact SO(8) is given in terms of fermionic ones. Short $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{OSp}(8|2,\mathbb{R})_R$ multiplets are constructed by taking the vacuum $|0\rangle_m = |0\rangle_L \times |0\rangle_R$ as the ground state and acting with creation operators. This corresponds to table 6 (cf. [22, table 11]). Tables 7–10 in contrast sit in $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{OSp}(8|2,\mathbb{R})_R$ long multiplets obtained from the ground states $(a_L^\dagger)^s|0\rangle_L \times (\xi_R^A)^s|0\rangle_R$, $s = 1, \ldots, 4$, however, these tables show only the primary states satisfying the lower bound $E_0 + \frac{1}{2}q = q_{\mathrm{hws}}$. E.g. the content of table 10 for $n = 0$ corresponds to the single primary satisfying the bound in the long multiplet given in table 9 of [22]. Here, it corresponds to a pure gauge mode of the graviton.

lowest state	q	SO(8)	E_0	s_0
$\|\Omega\rangle$	$n+2$	$(n{+}2000)$	$\frac{n}{2}+1$	0
$Q\|\Omega\rangle$	$n+\frac{3}{2}$	$(n{+}1010)$	$\frac{n}{2}+\frac{3}{2}$	$\frac{1}{2}$
$Q^2\|\Omega\rangle$	$n+1$	$(n100)$	$\frac{n}{2}+2$	1
$Q^3\|\Omega\rangle$	$n+\frac{1}{2}$	$(n001)$	$\frac{n}{2}+\frac{5}{2}$	$\frac{3}{2}$
$Q^4\|\Omega\rangle$	n	$(n000)$	$\frac{n}{2}+3$	2

Table 6: Supermultiplet with h.w.s. $|\,\Omega\,\rangle = |0\rangle_{n+2}$, cf. [22, table 11]. The SO(8) content at $n = 0$ is $35_v + 56_s + 28 + 8_s + 1$.

lowest state	q	SO(8) IIA	SO(8) IIB	E_0	s_0
$\|\Omega\rangle$	$n+\frac{3}{2}$	$(n{+}1010)$	$(n{+}1001)$	$\frac{n}{2}+\frac{3}{2}$	$-\frac{1}{2}$
$Q\|\Omega\rangle$	$n+1$	$(n020)+(n100)$	$(n{+}1000)+(n011)$	$\frac{n}{2}+2$	0
$Q^2\|\Omega\rangle$	$n+\frac{1}{2}$	$(n001)+(n-1110)$	$(n010)+(n-1101)$	$\frac{n}{2}+\frac{5}{2}$	$\frac{1}{2}$
$Q^3\|\Omega\rangle$	n	$(n000)+(n-1011)$	$(n-1100)+(n-1002)$	$\frac{n}{2}+3$	1
$Q^4\|\Omega\rangle$	$n-\frac{1}{2}$	$(n-1010)$	$(n-1001)$	$\frac{n}{2}+\frac{7}{2}$	$\frac{3}{2}$

Table 7: Supermultiplet with h.w.s. $|\,\Omega\,\rangle = \bar{Q}|0\rangle_{n+2} = a_L^\dagger|0\rangle_L \times \xi_R^A|0\rangle_R$ $(|\,\Omega\,\rangle = \widetilde{Q}|0\rangle_{n+2})$ for type IIA (IIB). The SO(8) content at $n=0$ is $56_s+35_c+28+8_s+1$ $(56_c+8_v+56_v+8_c)$.

lowest state	q	SO(8)	E_0	s_0
$\|\Omega\rangle$	$n+1$	$(n100)$	$\frac{n}{2}+2$	-1
$Q\|\Omega\rangle$	$n+\frac{1}{2}$	$(n001)+(n-1110)$	$\frac{n}{2}+\frac{5}{2}$	$-\frac{1}{2}$
$Q^2\|\Omega\rangle$	n	$(n000)+(n-1011)+(n-2200)$	$\frac{n}{2}+3$	0
$Q^3\|\Omega\rangle$	$n-\frac{1}{2}$	$(n-2101)+(n-1010)$	$\frac{n}{2}+\frac{7}{2}$	$\frac{1}{2}$
$Q^4\|\Omega\rangle$	$n-1$	$(n-2100)$	$\frac{n}{2}+4$	1

Table 8: Supermultiplet with h.w.s. $|\,\Omega\,\rangle = \bar{Q}^2|0\rangle_{n+2} = (a_L^\dagger)^2|0\rangle_L \times \xi_R^A\xi_R^B|0\rangle_R$. The SO(8) content at $n=0$ is $28+8_s+1$ and it is associated to gauge degrees of freedom coming from the massless vector, gravitino and graviton respectively.

lowest state	q	SO(8) IIA	SO(8) IIB	E_0	s_0
$\|\Omega\rangle$	$n+\frac{1}{2}$	$(n001)$	$(n010)$	$\frac{n}{2}+\frac{5}{2}$	$-\frac{3}{2}$
$Q\|\Omega\rangle$	n	$(n000)+(n-1011)$	$(n-1020)+(n-1100)$	$\frac{n}{2}+3$	-1
$Q^2\|\Omega\rangle$	$n-\frac{1}{2}$	$(n-1010)+(n-2101)$	$(n-1001)+(n-2110)$	$\frac{n}{2}+\frac{7}{2}$	$-\frac{1}{2}$
$Q^3\|\Omega\rangle$	$n-1$	$(n-2100)+(n-2002)$	$(n-1000)+(n-2011)$	$\frac{n}{2}+4$	0
$Q^4\|\Omega\rangle$	$n-\frac{3}{2}$	$(n-2001)$	$(n-2010)$	$\frac{n}{2}+\frac{9}{2}$	$\frac{1}{2}$

Table 9: Supermultiplet with h.w.s. $|\,\Omega\,\rangle = \bar{Q}^3|0\rangle_{n+2} = (a_L^\dagger)^3|0\rangle_L \times \xi_R^A\xi_R^B\xi_R^C|0\rangle_R$ $(|\,\Omega\,\rangle = \widetilde{Q}^3|0\rangle_{n+2})$ for type IIA (IIB). The content at $n=0$ is pure gauge in the SO(8) representations 8_s+1 (8_c).

lowest state	q	SO(8)	E_0	s_0
$\|\Omega\rangle$	n	$(n000)$	$\frac{n}{2}+3$	-2
$Q\|\Omega\rangle$	$n-\frac{1}{2}$	$(n-1010)$	$\frac{n}{2}+\frac{7}{2}$	$-\frac{3}{2}$
$Q^2\|\Omega\rangle$	$n-1$	$(n-2100)$	$\frac{n}{2}+4$	-1
$Q^3\|\Omega\rangle$	$n-\frac{3}{2}$	$(n-2001)$	$\frac{n}{2}+\frac{9}{2}$	$-\frac{1}{2}$
$Q^4\|\Omega\rangle$	$n-2$	$(n-2000)$	$\frac{n}{2}+5$	0

Table 10: Supermultiplet with h.w.s. $|\,\Omega\,\rangle = \bar{Q}^4|0\rangle_{n+2} = (a_L^\dagger)^4|0\rangle_L \times \xi_R^A\xi_R^B\xi_R^C\xi_R^D|0\rangle_R$. There is a pure gauge SO(8) singlet at $n=0$.

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lowest state	q	SO(8)	E_0	s_0
$\lvert\Omega\rangle$	$n+1$	$(n{+}1000)$	$\frac{n}{2}+\frac{3}{2}$	1
$Q\lvert\Omega\rangle$	$n+\frac{1}{2}$	$(n010)$	$\frac{n}{2}+2$	$\frac{1}{2}$
$Q^2\lvert\Omega\rangle$	n	$(n-1100)$	$\frac{n}{2}+\frac{5}{2}$	0
$Q^3\lvert\Omega\rangle$	$n-\frac{1}{2}$	$(n-1001)$	$\frac{n}{2}+3$	$\frac{1}{2}$
$Q^4\lvert\Omega\rangle$	$n-1$	$(n-1000)$	$\frac{n}{2}+\frac{7}{2}$	1

Table 11: Type I gauge supermultiplet with h.w.s. $\lvert\Omega\rangle = (a^\dagger)^2\,\lvert 0\rangle_L \times \lvert 0\rangle_R$. The SO(8) content at $n=0$ is $n_v\,(8_v+8_s)$.

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